COMPOSITIONALITY AND LOGIC IN LANGUAGE

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COMPOSITIONALITY & MEANING

Sentence-level compositionality & usage

A unicorn drinks tea

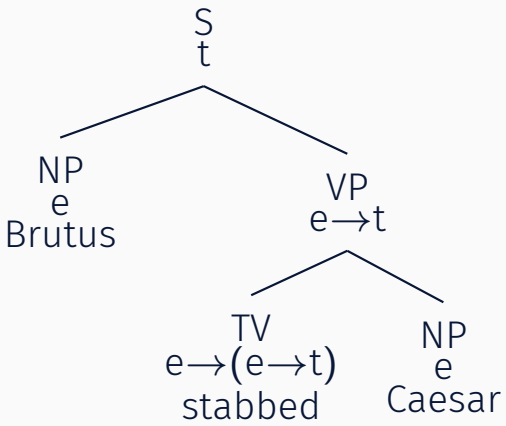
Sentence-level compositionality & usage?

Tea drinks a unicorn

Compositionality and meaning, a definition

"The meaning of an expression is a function of the meanings of its parts and of the way they are syntactically combined" [Partee, 1984]

[Montague, 1973], compositional semantics



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$$[\![\text{Brutus stabbed Caesar}]\!] = ([\![\text{stabbed}]\!][\![\text{Caesar}]\!])[\![\text{Brutus}]\!]$$

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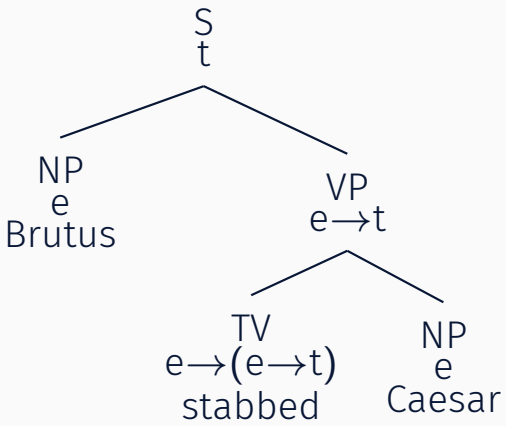

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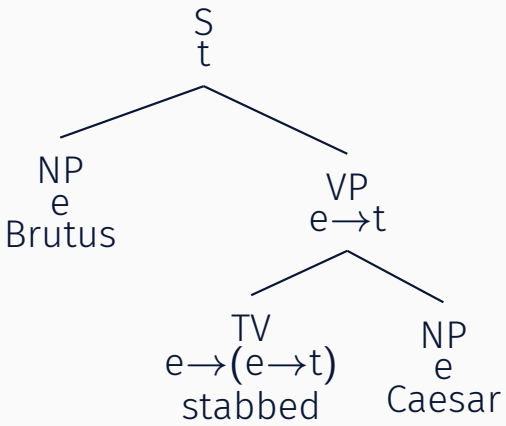

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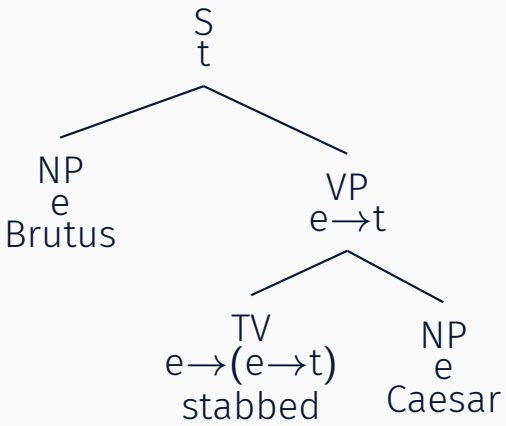

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NNS, REASONING, AND COMPOSITIONALITY

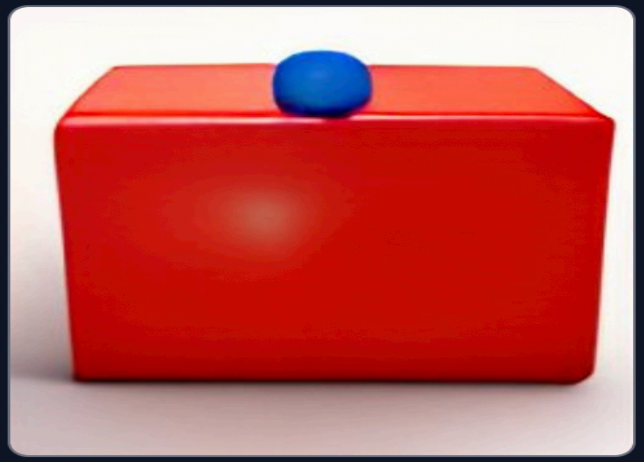
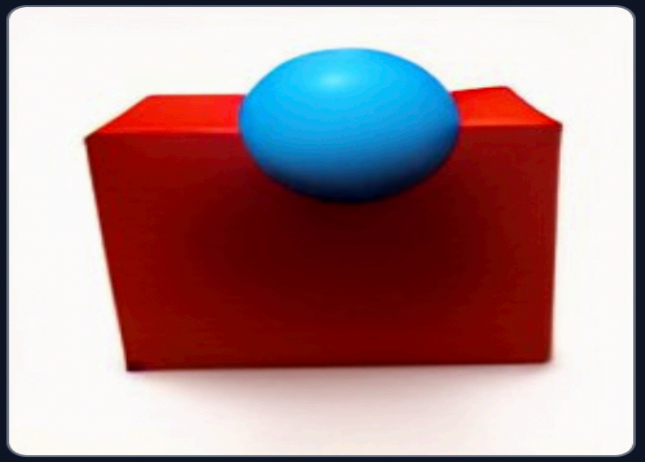
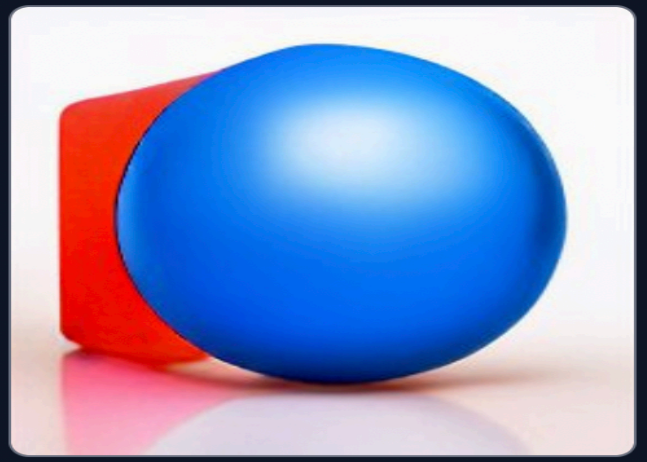
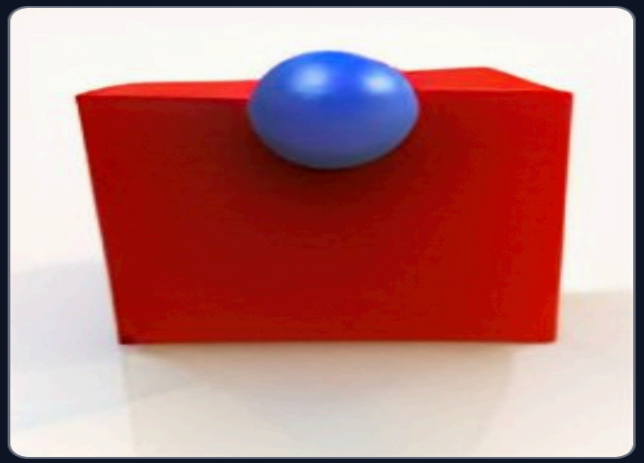
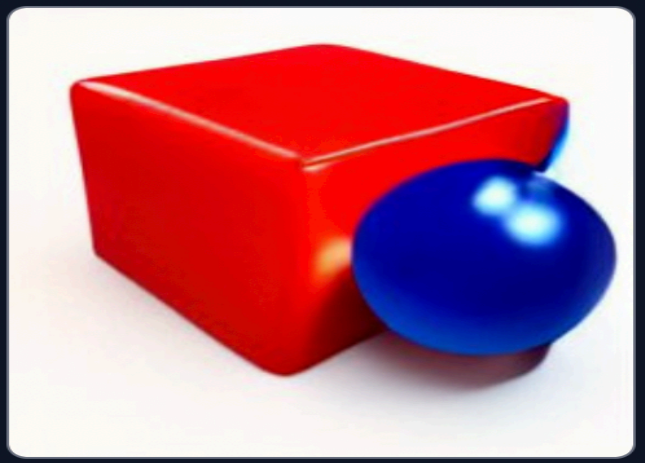
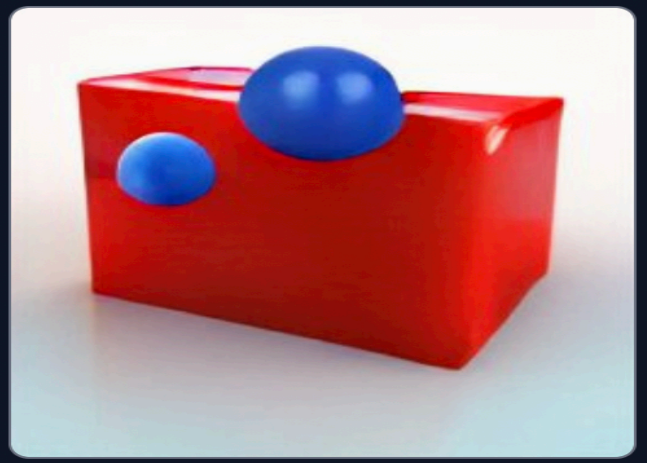
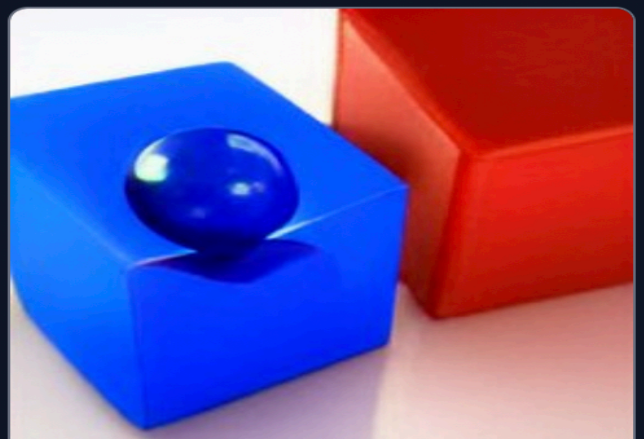
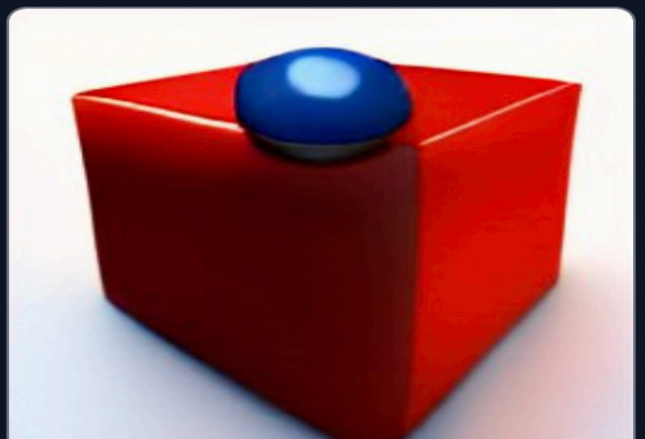
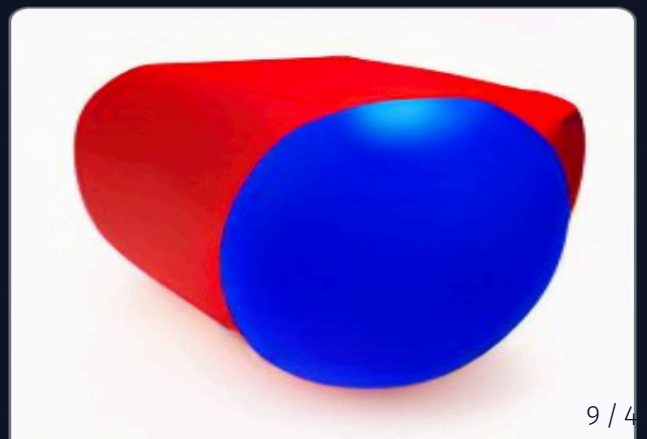
a unicorn drinking tea

Draw

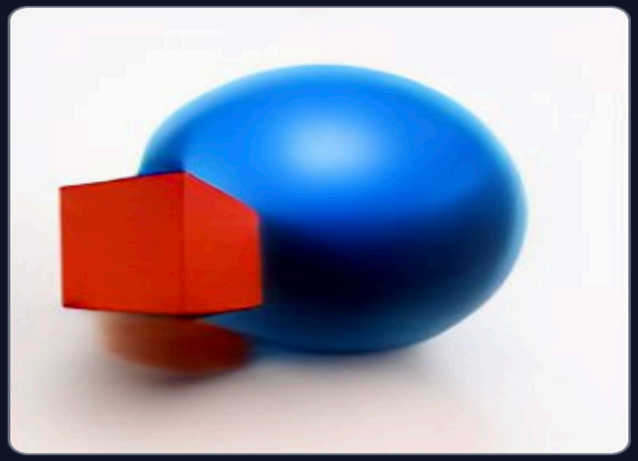
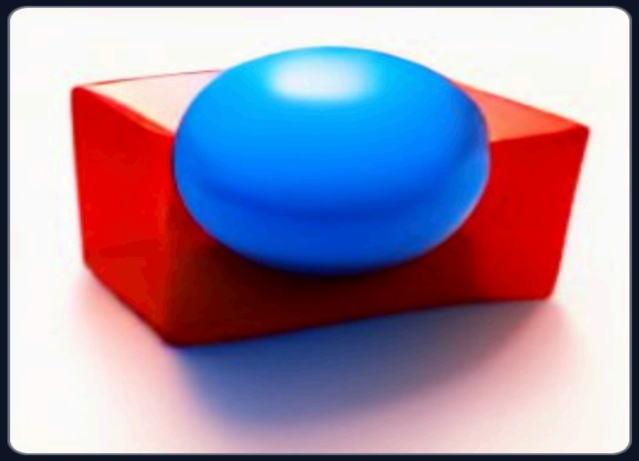
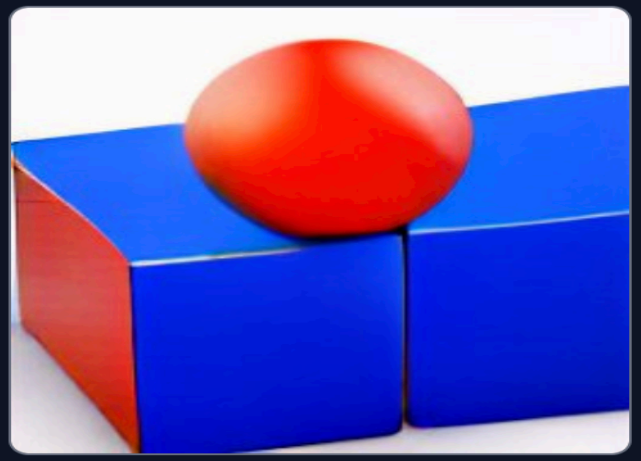
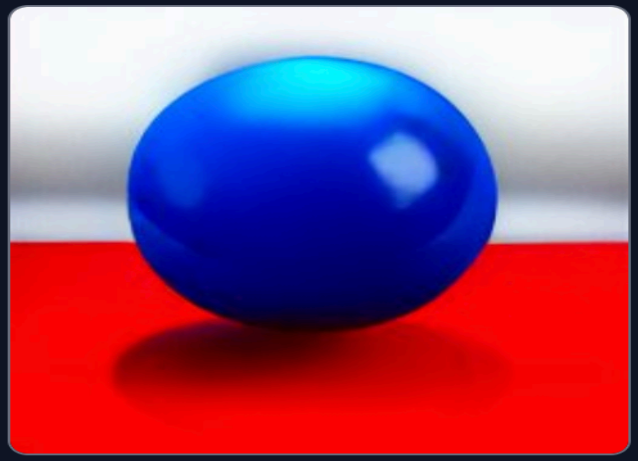
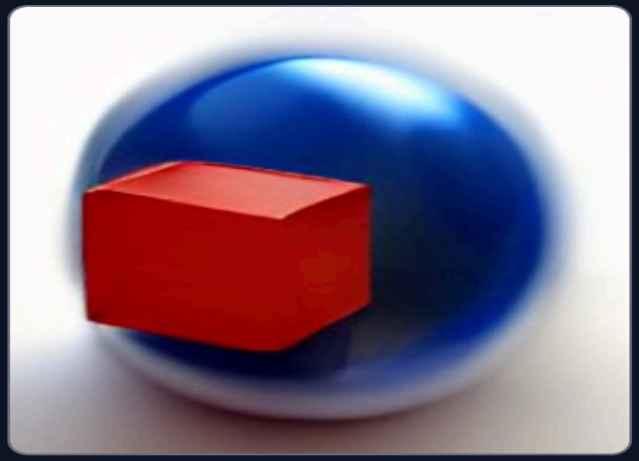
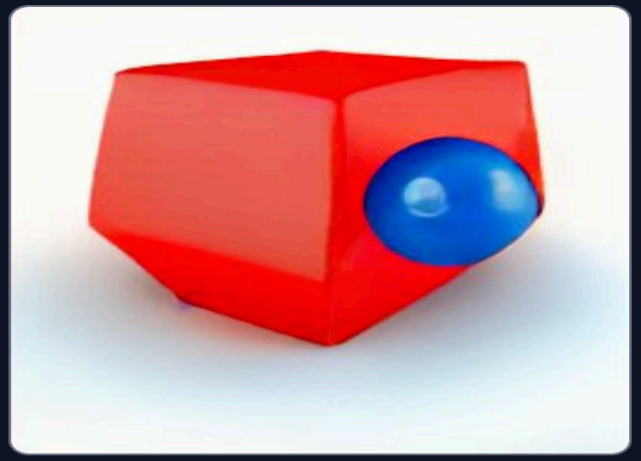
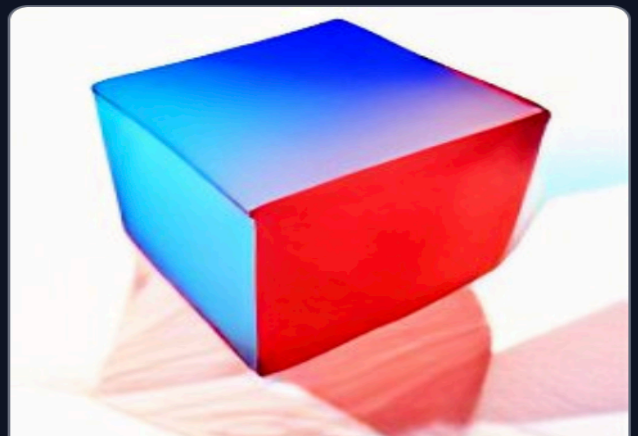
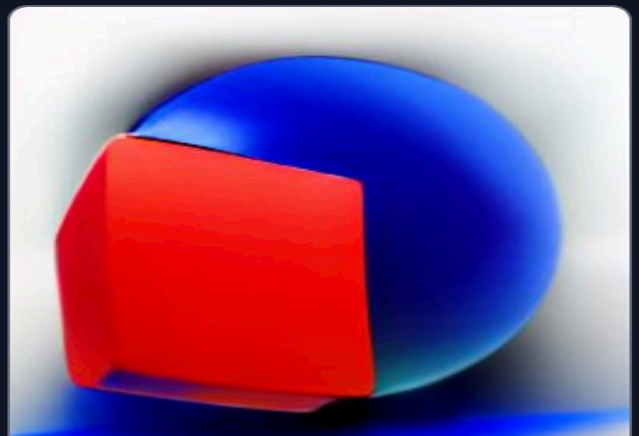
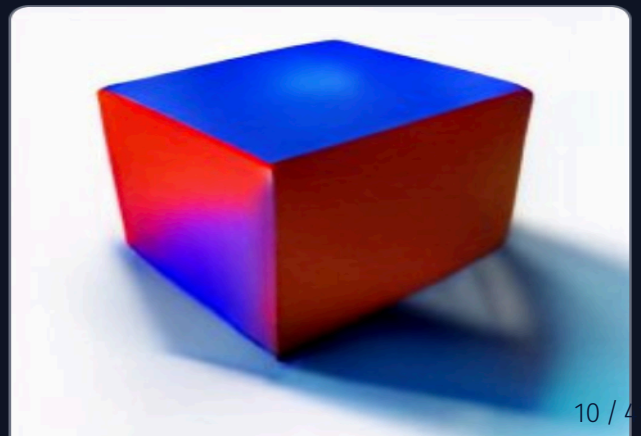
tea drinking a unicorn

Draw

blue ball behind a red cube

red cube behind a blue ball

Neural Networks and compositionality

In the past years, investigation of capacity of (neural) language models to use/produce/deduce compositional rules/sentences.

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- **Tree-structured composition in neural networks without tree-structured architectures**, [Bowman et al., 2015]
- **Diagnostic classifiers revealing how neural networks process hierarchical structure**, [Veldhoen et al., 2016]
- **Siamese recurrent networks learn first-order logic reasoning and exhibit zero-shot compositional generalization**, [Mul and Zuidema, 2019]

Neural Networks and compositionality

- **Analysing mathematical reasoning abilities of neural models**, [Saxton et al., 2019]
- **Compositionality decomposed: How do neural networks generalise?**, [Hupkes et al., 2020]
- **LogicInference: A New Dataset for Teaching Logical Inference to seq2seq Models**, [Ontanon et al., 2022]
- **Unit Testing for Concepts in Neural Networks**, [Lovering and Pavlick, 2022]

S. Bowman, S. R., Manning, C. D., & Potts, C. (2015). **Tree-structured composition in neural networks without tree-structured architectures.** arXiv preprint arXiv:1506.04834.

- ➔ Test whether neural sequence models such as LSTMs are able to discover and implicitly use recursive compositional structures (with explicit cues).
- ➔ Results: tree models perform significantly better than LSTMs on such tasks.
- ➔ Classification task: what type of logical relation (out of 7 possible) between pairs of sentences.

Saxton, D., Grefenstette, E., Hill, F., & Kohli, P. (2019). **Analysing mathematical reasoning abilities of neural models.** arXiv preprint arXiv:1904.01557.

- ➔ Task suite of mathematical problems (arithmetic, algebra, probability, calculus) in English, question in English, short answer: only number, yes, no.
- ➔ Designed to test the capacity of NNs (Transformers and Recurrent architectures) to compose mathematical concepts and generalize mathematical operations.
- ➔ Performance varies greatly from one type of problem to another.

Hupkes, D., Dankers, V., Mul, M., & Bruni, E. (2020). **Compositionality decomposed: How do neural networks generalise?** Journal of Artificial Intelligence Research, 67, 757-795.

➔ Set of 5 tests for NNs capacity to compositionally generalize:

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➔ Set of 5 tests for NNs capacity to compositionally generalize:

(1) can they successfully recombine known parts and rules?

(2) can they extend their predictions beyond the length they've seen in training?

(3) do they compose in a local or in a global way?

(4) are their predictions robust to synonym substitution?

(5) do they favor rules or exceptions during training?

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➔ Instantiation of the 5 tests on an artifical dataset: PCFG SET, and apply them to a recurrent NN, a convolution-based NN and a transformer.

In our work [Guzmán et al., 2022]

We would like to investigate the capacity of neural networks (NNs) to learn compositional structures/rules.

- ➔ In a minimal setting: **simple corpus**, where all composition comes from the structure of proofs.
- ➔ Testing different aspects of compositionality.

Compositionality, simple corpus, and NNs

Given that a model can derive:

Premisses: "A $\rightarrow$ B" "B $\rightarrow$ C"

Conclusion: "A $\rightarrow$ C"

Then it should derive:

Premisses: "A $\rightarrow$ B" "B $\rightarrow$ C" "C $\rightarrow$ D"

Conclusion: "A $\rightarrow$ D"

Compositionality, simple corpus, and NNs

Given that a model can derive:

Premisses: “A $\rightarrow$ B” “B $\rightarrow$ C”

Conclusion: “A $\rightarrow$ C”

Then it should derive:

Premisses: “X $\rightarrow$ Y” “Y $\rightarrow$ Z”

Conclusion: “X $\rightarrow$ Z”

A SIMPLE CORPUS

Corpus description

Constants: $\mathcal{C} = \{X_1 \dots X_n\}$

Formulas: "$X_i \to X_j$"

Derivation rule:

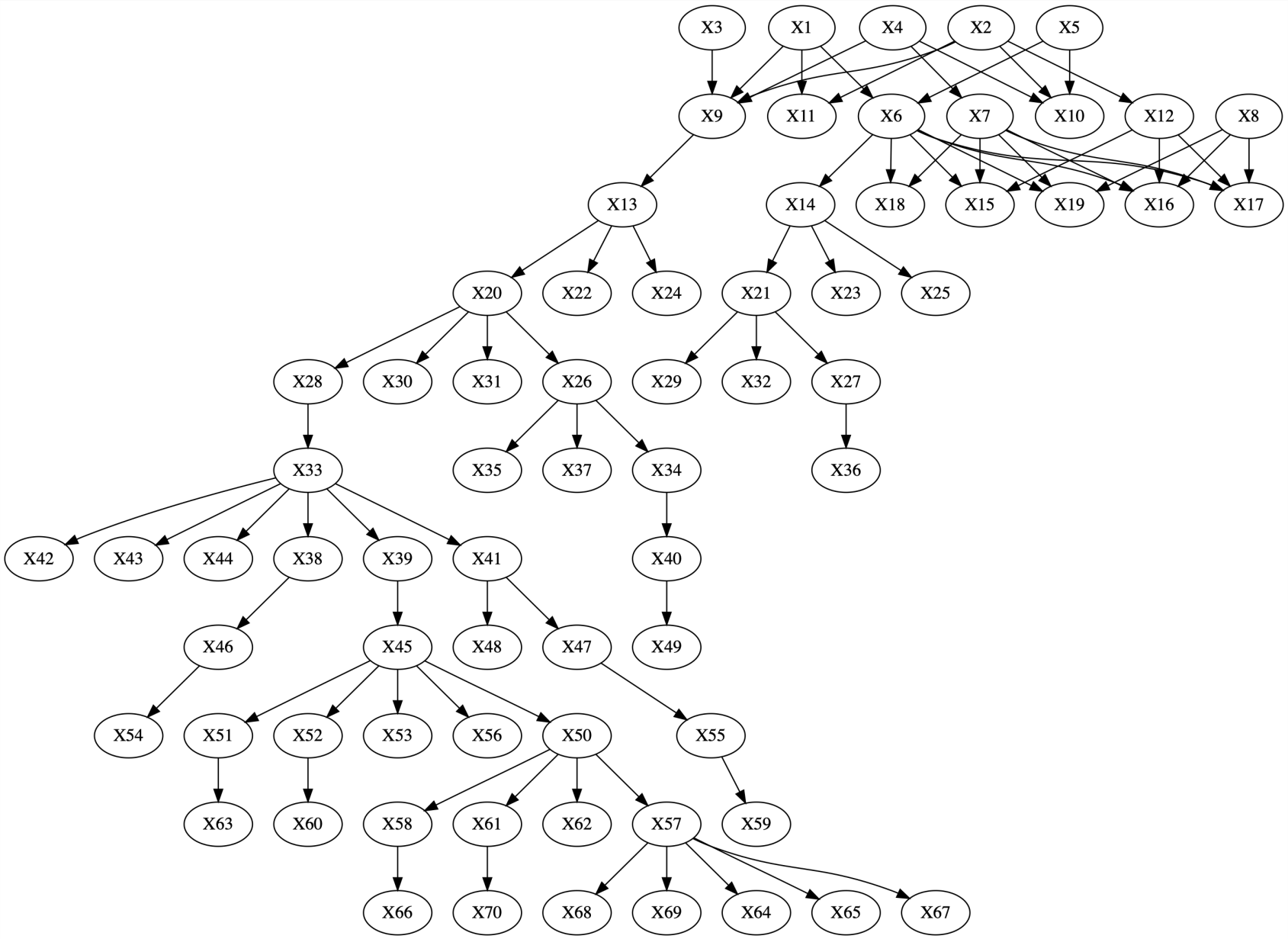
$$\frac{X_i \to X_j \qquad X_j \to X_k}{X_i \to X_k}$$

Example

$$\{X_1 \to X_2,\ X_2 \to X_3,\ X_3 \to X_4,\ X_4 \to X_5\} \vdash X_1 \to X_5$$

$$\dfrac{\dfrac{\dfrac{(1)\ X_1 \to X_2 \qquad (2)\ X_2 \to X_3}{X_1 \to X_3} \qquad (3)\ X_3 \to X_4}{X_1 \to X_4} \qquad (4)\ X_4 \to X_5}{X_1 \to X_5}$$

Knowledge Base (KB)



Knowledge Base (KB)

Parameters of the $\mathcal{KB}$: # constants $= 70$, # premises $= 82$, # hypotheses $= 4,830$, # valid hypotheses $= 531$.

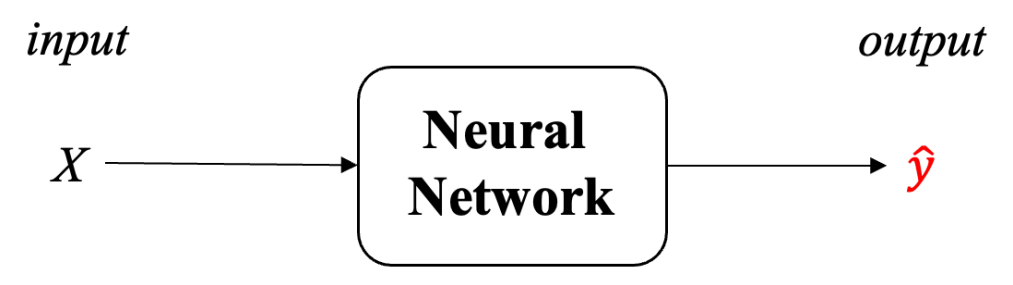
Length of proof	# of proofs
0	4,299
1	82
2	75
3	63
4	60
5	51
6	54
7	41
8	42
9	35
10	28

Connectionnist component

Given a knowledge base $\mathcal{KB}$ and a hypothesis $\mathbf{h}$, our connectionist component is a neural network that selects the (most likely) set of formulas from $\mathcal{KB}$ that are necessary to derive $\mathbf{h}$.

- ➔ Architectures: Multilayer perceptron (MLP), Recurrent neural network (RNN)

Connectionnist component



input: X is a vector that encodes a set $P = \{p_1, p_2, \ldots, p_n\}$ of all premisses and a hypothesis h

output: $\hat{y}$ is a vector of size n such that for each $i \in [1..n]$

$$\hat{y}_i = \begin{cases} 1 & \text{if } p_i \in \pi \\ 0 & \text{otherwise} \end{cases}$$

Where $\pi \subset P$ and $\pi \vdash h$, and π is necessary to prove h.

Example

1. $X = [\text{“}X_1 \rightarrow X_3\text{”} \text{“}X_3 \rightarrow X_6\text{”} \text{“}X_6 \rightarrow X_4\text{”} \text{“}X_2 \rightarrow X_7\text{”} \text{“}X_2 \rightarrow X_4\text{”} \text{“}X_1 \rightarrow X_4\text{”}]$

   $\hat{y} = [1 \quad 1 \quad 1 \quad 0 \quad 0]$

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2. $X = [\text{“}X_1 \rightarrow X_3\text{”} \text{“}X_3 \rightarrow X_6\text{”} \text{“}X_6 \rightarrow X_4\text{”} \text{“}X_2 \rightarrow X_7\text{”} \text{“}X_2 \rightarrow X_4\text{”} \text{“}X_1 \rightarrow X_7\text{”}]$

   $\hat{y} = [0 \quad 0 \quad 0 \quad 0 \quad 0]$

EXPERIMENTS AND RESULTS

Initial experiment – data distribution

Length of proofs	Total data	Train data	Test data
0	4299	3224	1075
1	82	62	20
2	75	56	19
3	63	47	16
4	60	45	15
5	51	38	13
6	54	40	14
7	41	31	10
8	42	32	10
9	35	26	9
10	28	21	7

Table: 75/25% split of the $\mathcal{KB}$, stratified by length of proofs

Initial experiment – accuracies

Length of proofs	MLP test	RNN test
0	99.91%	99.81%
1	40.00%	50.00%
2	57.89%	63.16%
3	68.75%	75.00%
4	93.33%	93.33%
5	100.00%	100.00%
6	100.00%	100.00%
7	100.00%	100.00%
8	100.00%	90.00%
9	100.00%	100.00%
10	100.00%	100.00%

Initial experiment – accuracies

|proofs|	MLP test	RNN test	hypothesis	# of sub-proofs	# of formulas
0	99.91%	99.81%	3224	0	0
1	40.00%	50.00%	62	62	62
2	57.89%	63.16%	56	138	194
3	68.75%	75.00%	47	202	362
4	93.33%	93.33%	45	331	695
5	100.00%	100.00%	38	416	1017
6	100.00%	100.00%	40	640	1756
7	100.00%	100.00%	31	645	2002
8	100.00%	90.00%	32	839	2869
9	100.00%	100.00%	26	845	3178
10	100.00%	100.00%	21	827	3384

Hamming distances

Let y be the expected output vector for our NN, $\hat{y}$ the actual output vector:

$$y = [1\ 1\ 0\ 0\ 1]$$
$$\hat{y} = [0.68\ 0.98\ 0.33\ 0.12\ 0.46]$$
$$\hat{y} \simeq [1\ 1\ 0\ 0\ 0]$$

Hamming distances

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$$y = [1\ 1\ 0\ 0\ 1]$$
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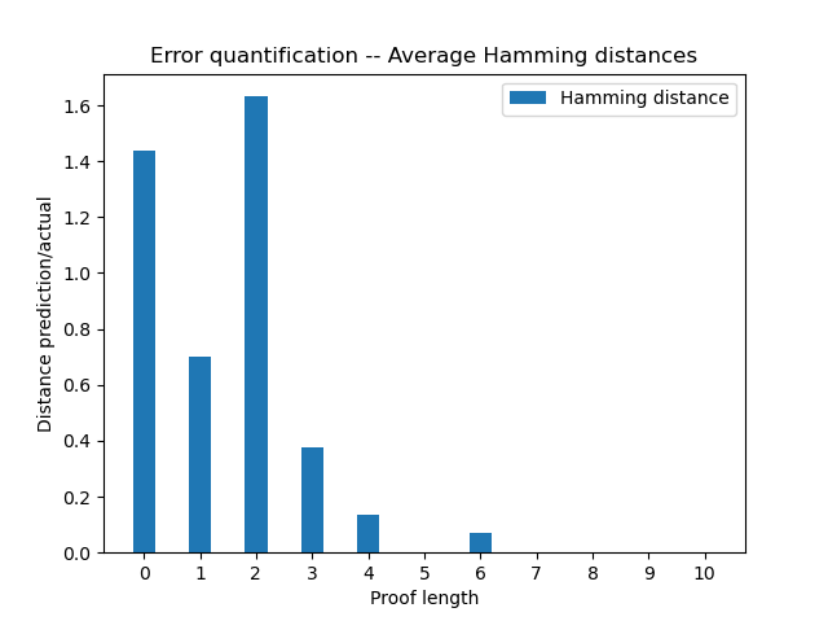
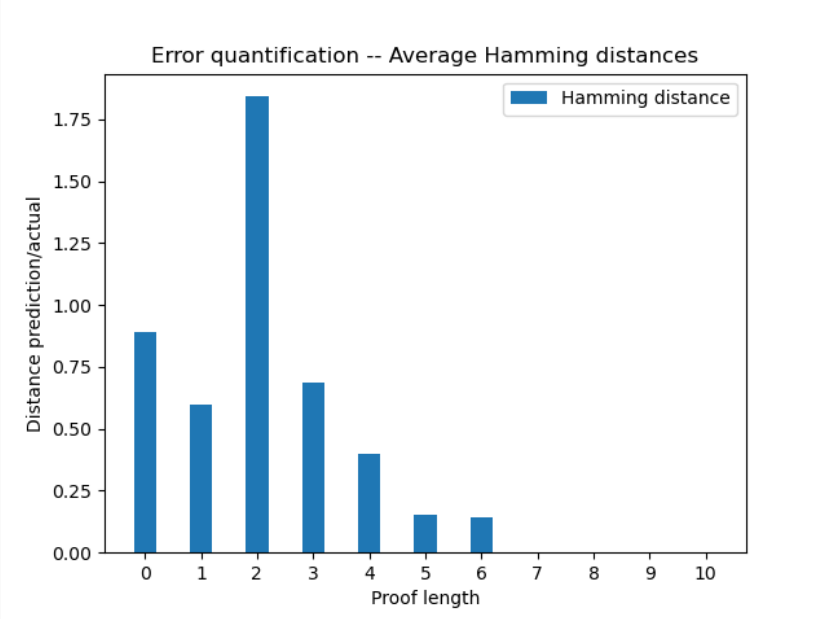
$$\hat{y} \simeq [1\ 1\ 0\ 0\ 0]$$

$$H(y, \hat{y}) = 1$$

Hamming distances – Initial experiment



(a) MLP



(b) RNN

Figure: Average Hamming distances between the expected output and the actual output for the initial experiment

Compositionality tests

We explored compositionality tests in the context of our simple corpus.

- ➔ Variations in the number of premisses needed to prove the conclusion:
  - Train on short proofs, test on long proofs
  - Train on long proofs, test on short proofs
- ➔ Permutations in the order of the constants

Variations in the number of premisses

The model is trained on proofs from length n_1 to n_2 ($n_1 < n_2$). Then, how does it perform when confronted to valid proofs, including proofs of length $n < n_1$ or $n > n_2$?

Variations in the number of premisses – results

Train data	MLP test	RNN test
0-9	100.0%	100.0%
0-8	92.06%	93.65%
0-7	73.33%	82.86%
0-6	45.89%	65.07%
0-5	17.0%	46.0%

(a) Unseen longer proofs

Train data	MLP test	RNN test
1-10	44.1%	48.08%
2-10	40.95%	41.61%
3-10	37.5%	40.13%
4-10	7.37%	11.06%
5-10	1.81%	2.16%

(b) Unseen shorter proofs

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➔ The less data is used for train, the less the model learns.

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(b) Unseen shorter proofs

- ➔ The less data is used for train, the less the model learns.
- ➔ The RNN has better accuracy than the MLP.

Variations in the number of premisses – results

Train data	MLP test	RNN test
0-9	100.0%	100.0%
0-8	92.06%	93.65%
0-7	73.33%	82.86%
0-6	45.89%	65.07%
0-5	17.0%	46.0%

(a) Unseen longer proofs

Train data	MLP test	RNN test
1-10	44.1%	48.08%
2-10	40.95%	41.61%
3-10	37.5%	40.13%
4-10	7.37%	11.06%
5-10	1.81%	2.16%

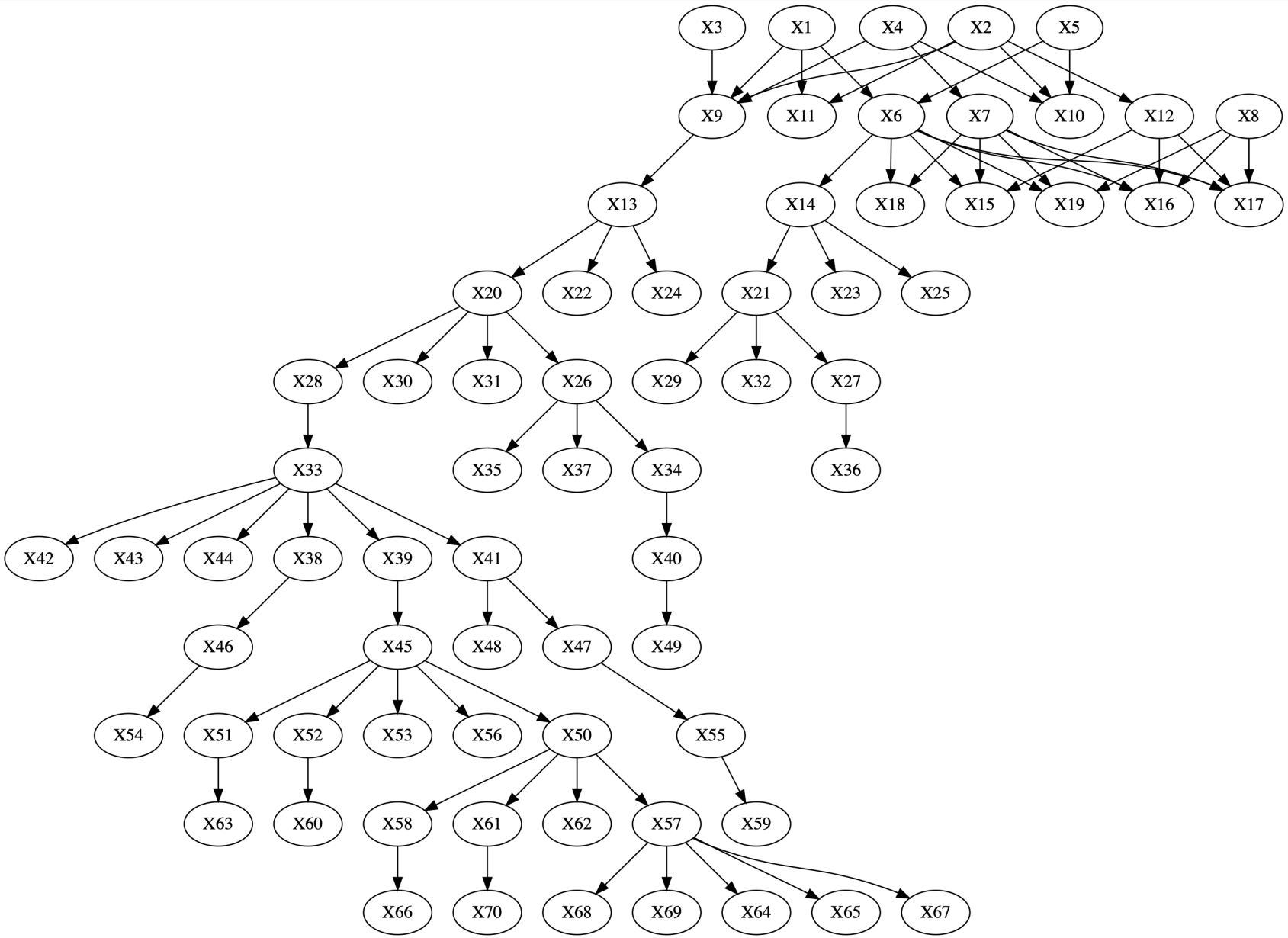
(b) Unseen shorter proofs

- ➔ The less data is used for train, the less the model learns.
- ➔ The RNN has better accuracy than the MLP.

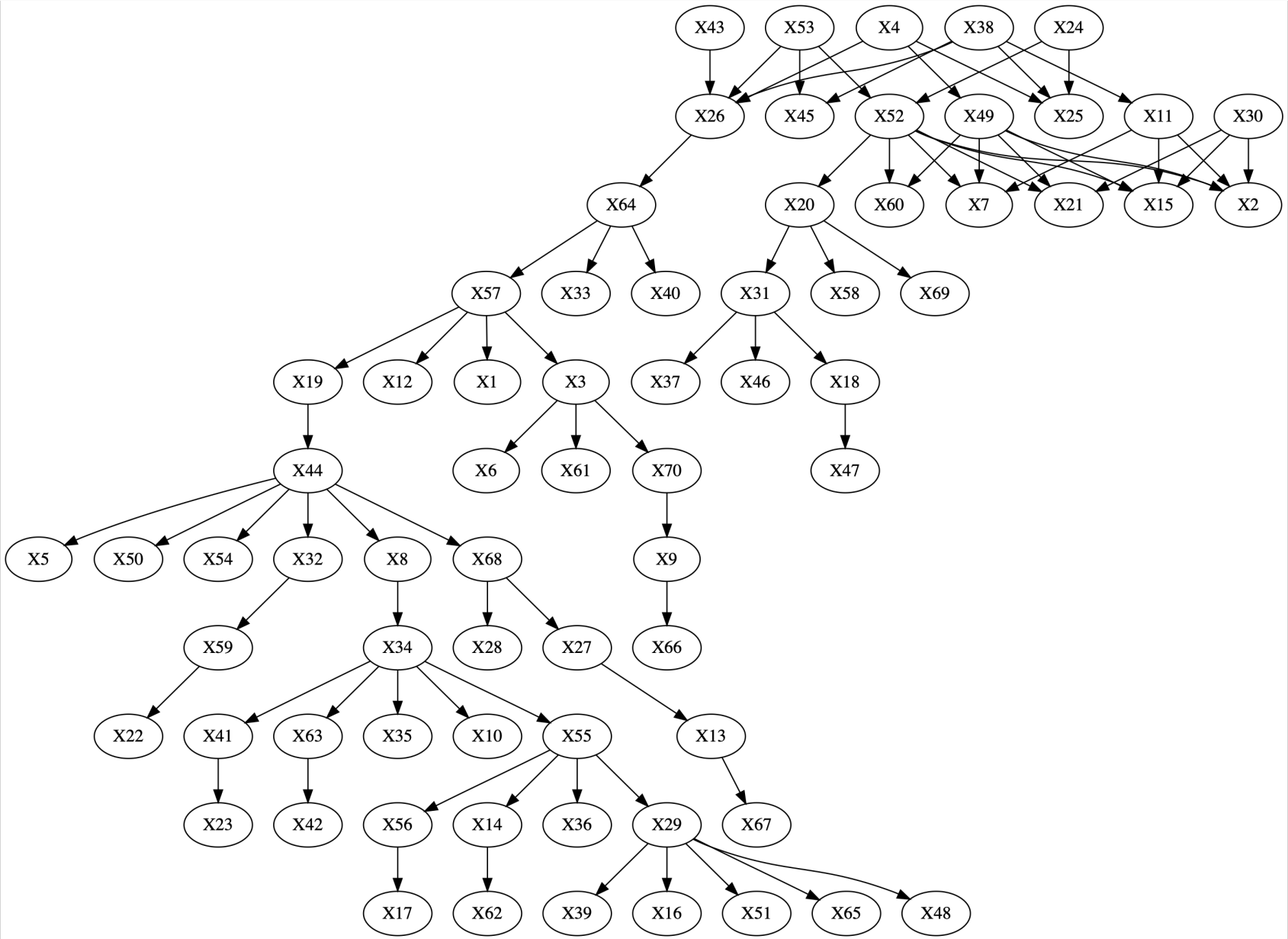
Permutations of constants

If the model has been trained on a given $\mathcal{KB}$, then a permutation function $\sigma : \mathcal{C} \to \mathcal{C}$ has been applied to $\mathcal{KB}$ to obtain $\mathcal{KB}'$, how will the model behave with inputs from $\mathcal{KB}'$?

Knowledge Base (KB)



Permuted Knowledge Base (KB')



Permutations of constants

Length of proofs	MLP test	RNN test
0	17.84%	66.43%
1	0.00%	0.00%
2	0.00%	1.33%
3	0.00%	0.00%
4	0.00%	0.00%
5	0.00%	0.00%
6	0.00%	0.00%
7	0.00%	0.00%
8	0.00%	0.00%
9	0.00%	0.00%
10	0.00%	0.00%

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6	0.00%	0.00%
7	0.00%	0.00%
8	0.00%	0.00%
9	0.00%	0.00%
10	0.00%	0.00%

➔ Next test: how does the model behave w.r.t permutation of constants once it has been exposed to permuted constants?

CONCLUSION

Conclusion & future work

NNs & compositionality:

- ➔ NNs pick up some **structure** from data: some amount of **generalization** in the variations in proof length compositionality tests, sub-proofs play a role in learning;
- ➔ Limited generalization: unseen length experiment, high sensitivity to the order of constants, $\gg$ overall structure of the $\mathcal{KB}$.

Conclusion & future work

NNs & compositionality:

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- ➔ Limited generalization: unseen length experiment, high sensitivity to the order of constants, $\gg$ overall structure of the $\mathcal{KB}$.

To be investigated next:

- ➔ what is the representation of the data that the NN builds in training?
- ➔ comparative studies through different dataset sizes, data encodings, other NN architectures.

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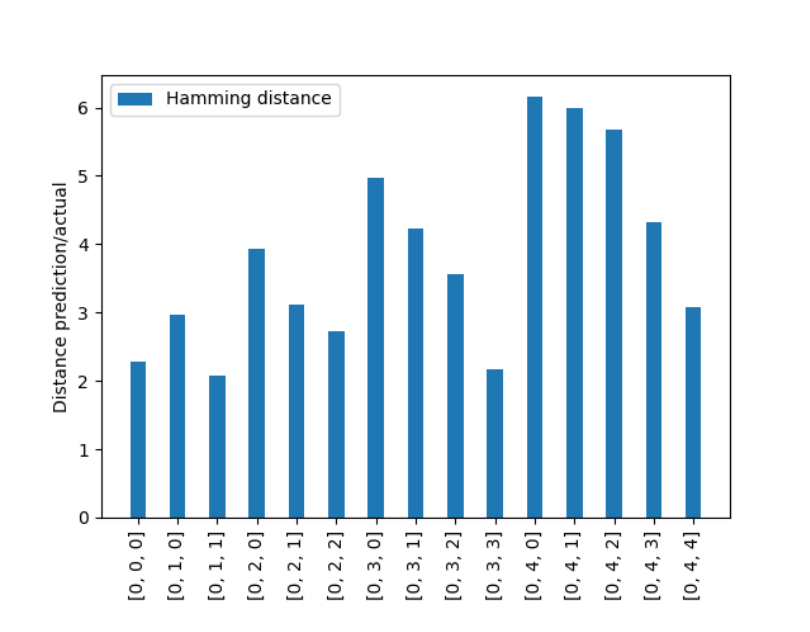
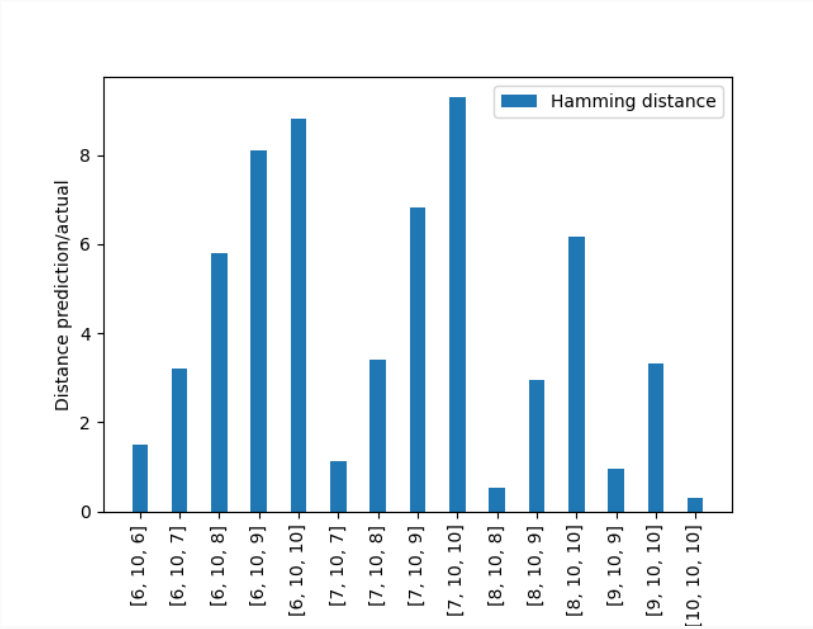
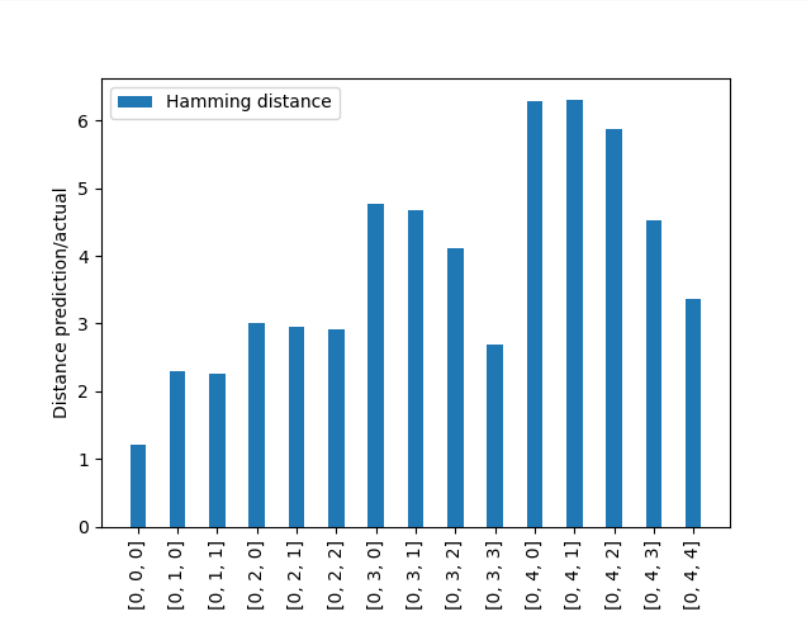


(a) Minimal length of unseen proof: 0

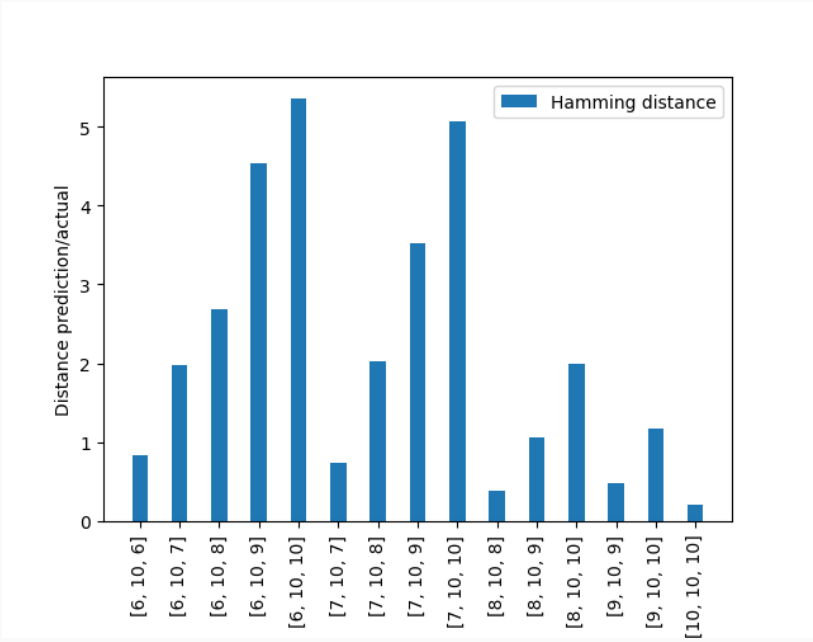


(b) Maximal length of unseen proof: 10

Figure: Average Hamming distances between the expected output and the actual output for the unseen length experiment, MLP



(a) Minimal length of unseen proof: 0



(b) Maximal length of unseen proof: 10

Figure: Average Hamming distances between the expected output and the actual output for the unseen length experiment, RNN

Data encoding

Each formula from input X is encoded in a vector of dimension 2n (where n is the the size of the set $\mathcal{C}$) as a one-hot fashion.

Example: let $\mathcal{C} = \{X_1, X_2, X_3, X_4, X_5\}$, then the formula $X_2 \rightarrow X_5$ is encoded as

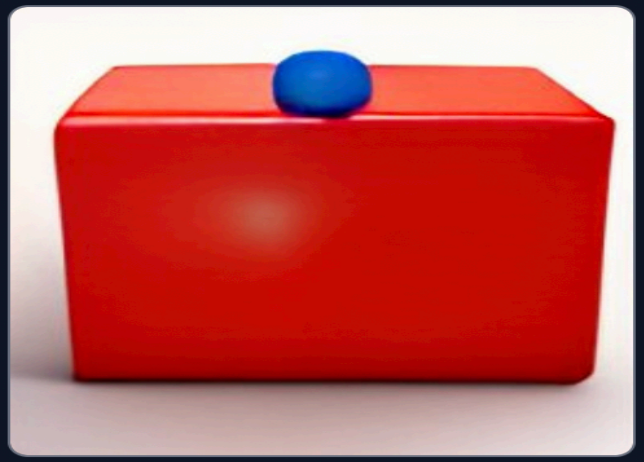
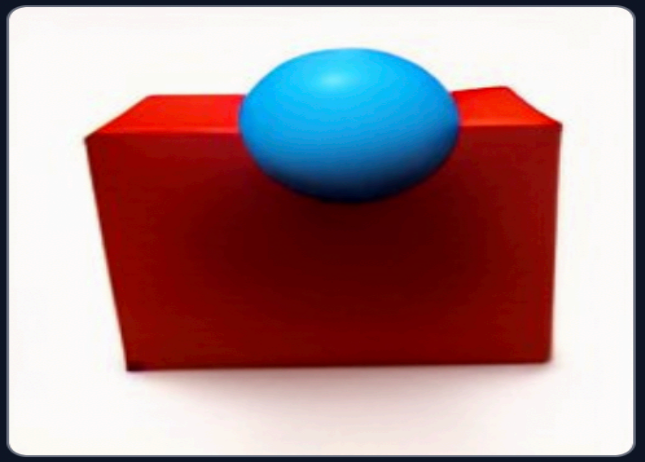
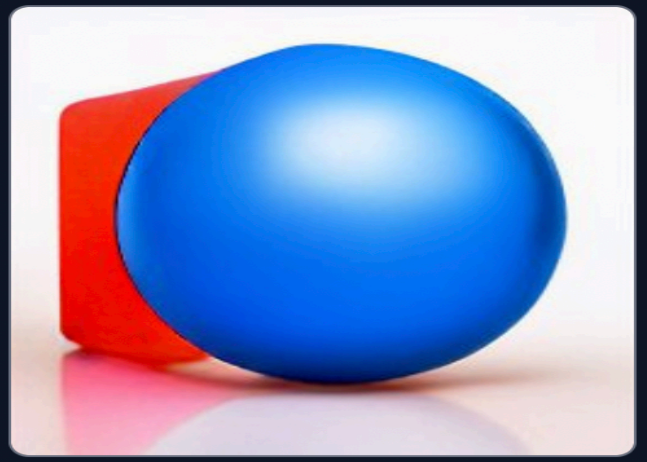
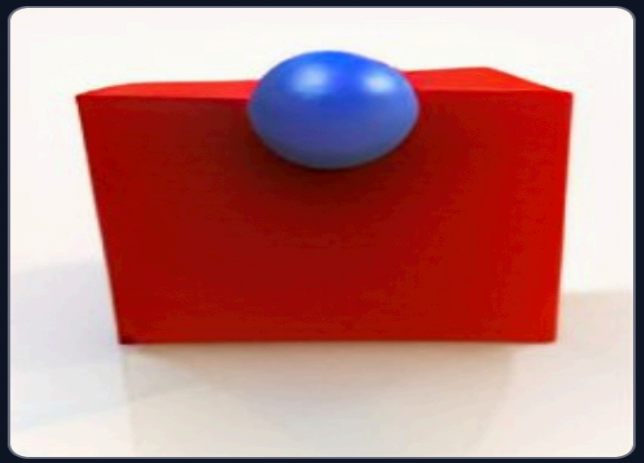
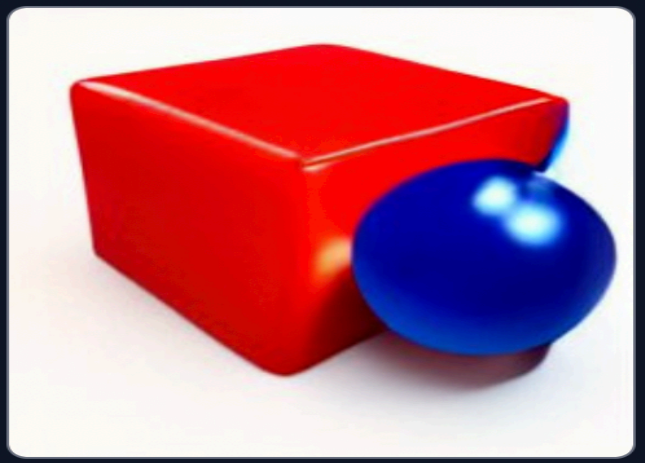
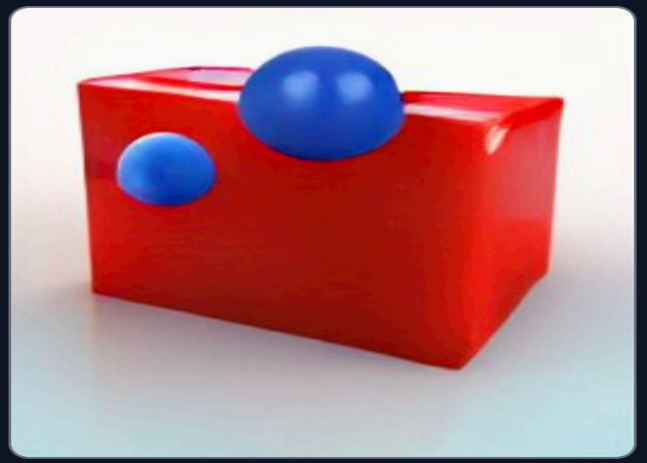
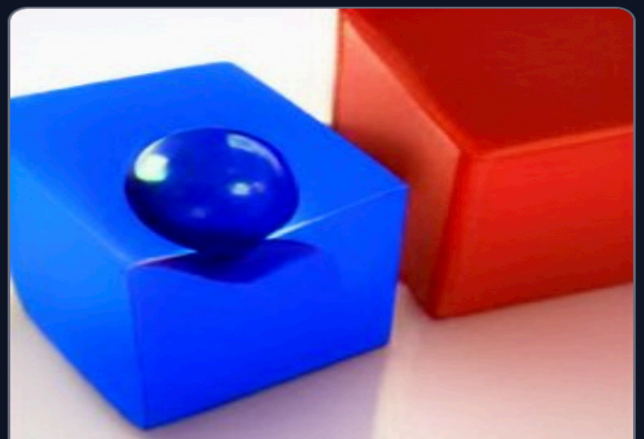
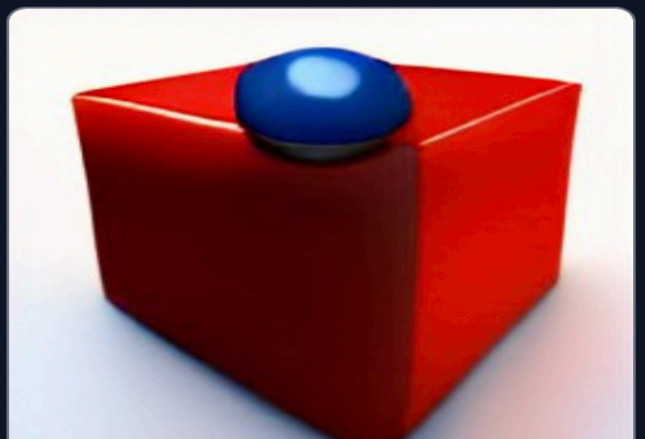
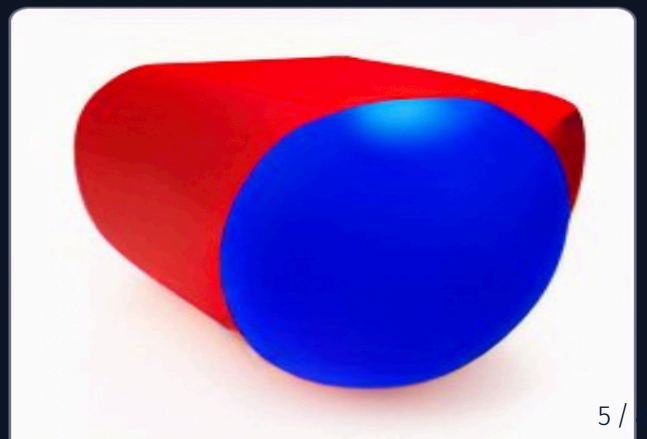
$$[0\ 1\ 0\ 0\ 0 \mid 0\ 0\ 0\ 0\ 1]$$

where the first n digits represent X_2 and the constant X_5 is encoded within the last n bits from the vector.

Neural networks setup

Architecture	MLP	RNN
Optimization algorithm	Adamax algorithm, default learning rate 0.001	same
# hidden layers	1	2
# neurons in each layer	2500	200
Function in hidden layers	hyperbolic tangent function (tanh)	tanh
Function in the output layer	sigmoid function	sigmoid function
# epochs	between 200 and 300 epochs	same
batch size	20	20

blue ball behind a red cube

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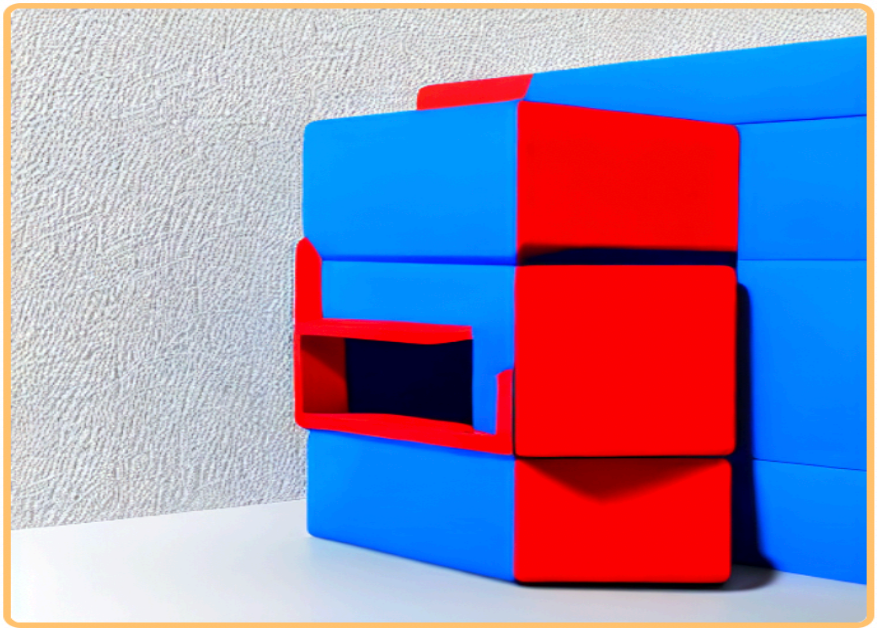
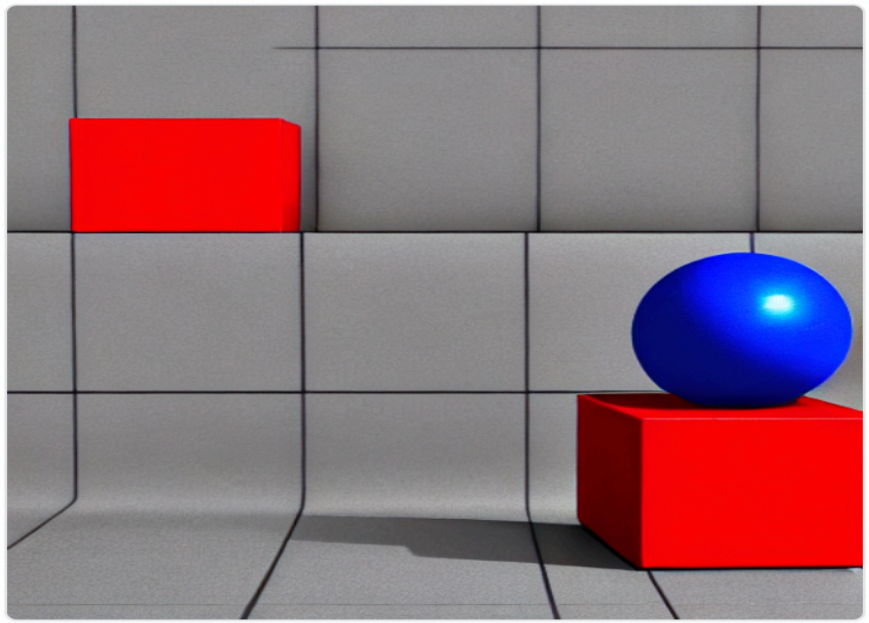
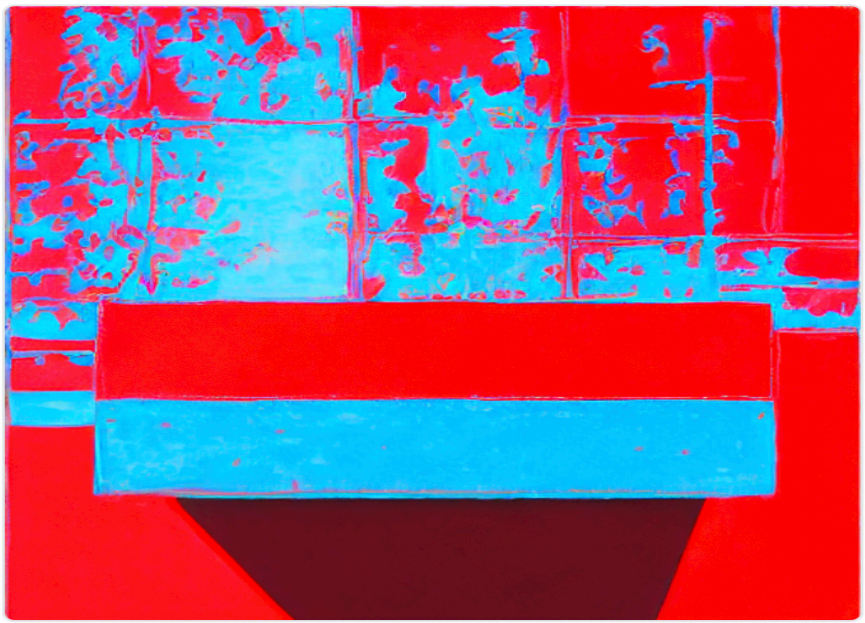
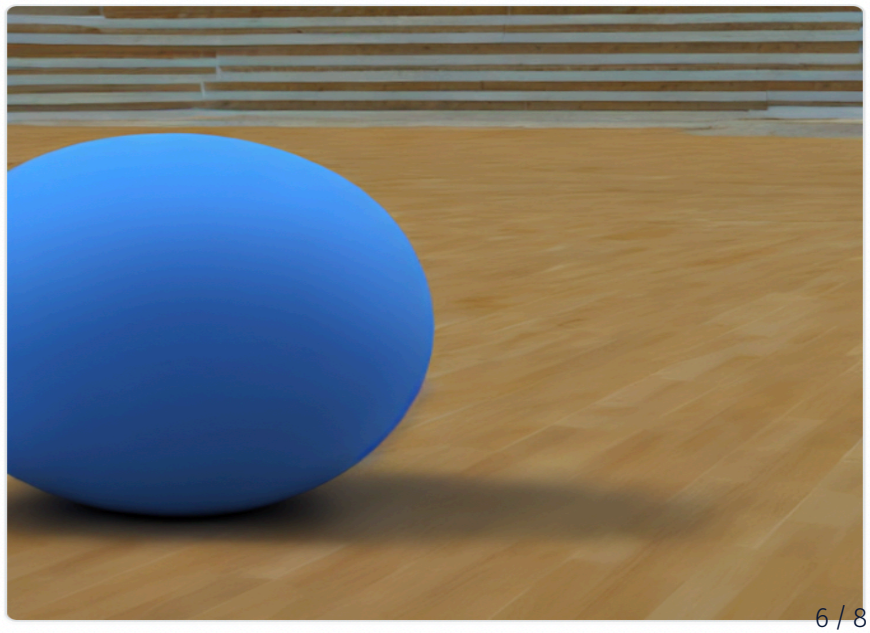
blue ball behind a red cube

Generate image

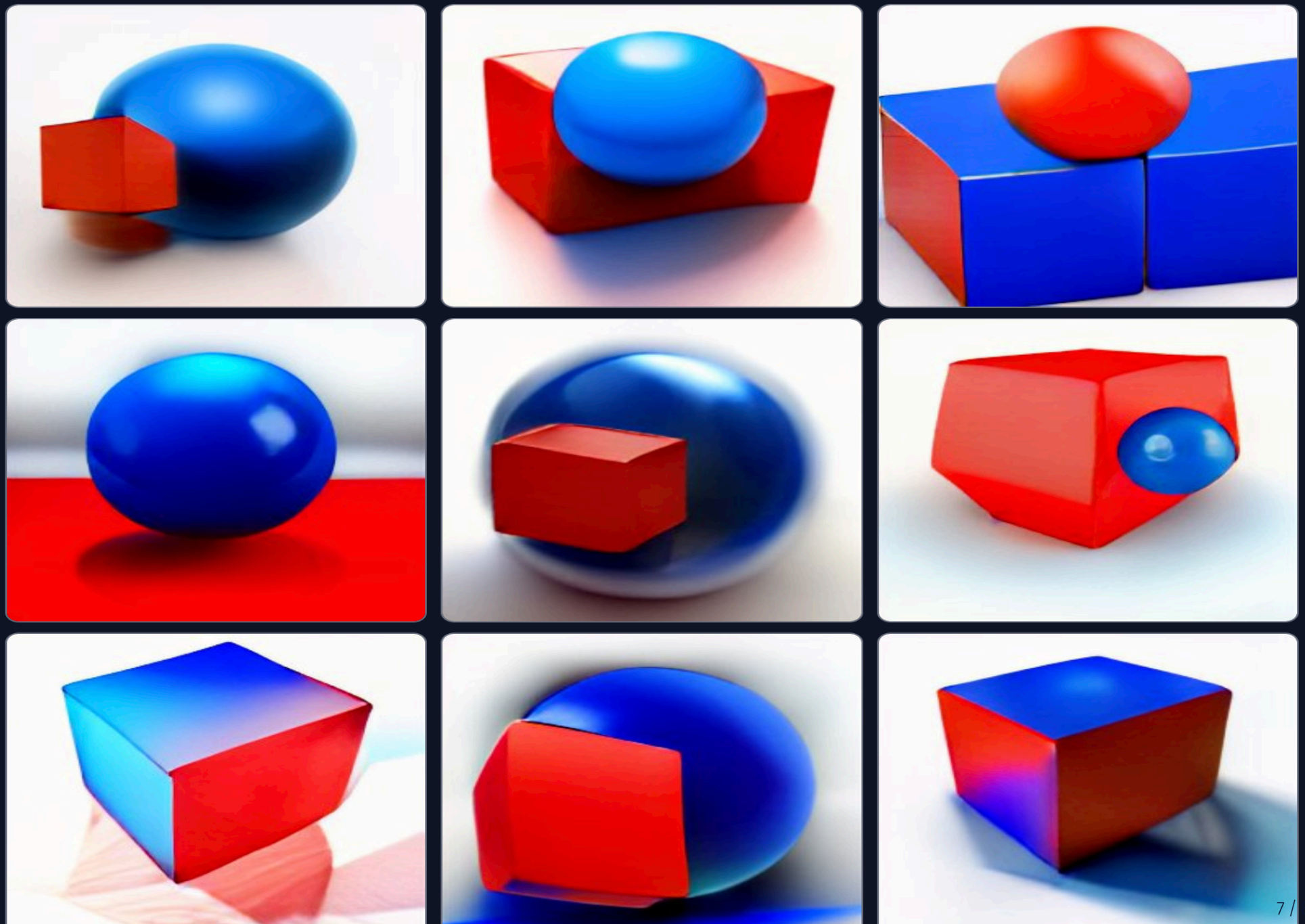








red cube behind a blue ball



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red cube behind a blue ball

Generate image